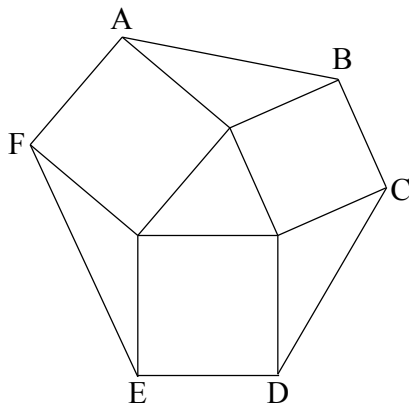


POMONA-WISCONSIN MATHEMATICS TALENT SEARCH

PROBLEM SET I (2008-2009)

OCTOBER 2008

- Let $p = 101010 \cdots 01$ be the m -digit number in which the first and last digits are 1 and the digits alternate between 1 and 0. For which positive integers m is the number p prime.
- In the figure on the right, hexagon $ABCDEF$ is divided into three squares and four triangles. Show that the areas of all four triangles are equal.
- Find all positive integers x and y such that $x \leq y \leq 2x$ and $1 + x^2 + y^2 = 3xy$.
- Your calculator will show that the number



$$\sqrt[3]{7 + 5\sqrt{2}} + \sqrt[3]{7 - 5\sqrt{2}}$$

is approximately an integer. Decide whether or not it is exactly an integer, and prove that your answer is correct.

- Let F_n be the n th Fibonacci number. Thus $F_1 = 1, F_2 = 1, F_3 = 2, F_4 = 3, F_5 = 5$, and in general for $n > 2$, we have $F_n = F_{n-1} + F_{n-2}$. Now for each integer $n \geq 1$, write $A_n = (F_{n+1})^2 - (F_n)^2 - F_n F_{n+1}$ and $B_n = (F_n)^2 + (F_{n+2})^2 - 3F_n F_{n+2}$. Find simple formulas for A_n and B_n and use them to compute A_{1000} and B_{1000} .

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

Return To	MATHEMATICS TALENT SEARCH Dept. of Mathematics, Pomona College 610 N. College Ave., Claremont, CA 91711-6348 mathsearch@pomona.edu	Deadline November 3, 2008	
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