

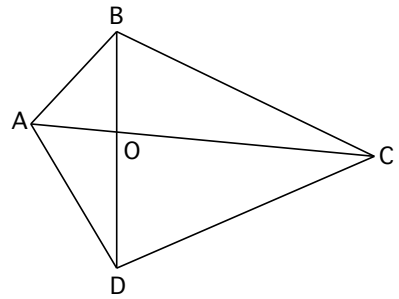
POMONA-WISCONSIN MATHEMATICS TALENT SEARCH

PROBLEM SET II (2007-2008)

NOVEMBER 2007

1. Let  $U$  be a set containing 29 objects and let  $S_1, S_2, \dots, S_{10}$  be 10 subsets of  $U$ , not necessarily distinct. Suppose that every 5 of the subsets taken together contain all 29 objects of  $U$ . Show that some three of these subsets taken together contain all 29 objects of  $U$ .

2. Let  $ABCD$  be the convex quadrilateral, as shown, and let  $O$  be the point of intersection of its two diagonals. Suppose the area of  $\triangle ABD$  is 1, the area of  $\triangle BCA$  is 2 and the area of  $\triangle DAC$  is 3. Find the areas of  $\triangle CDB$  and  $\triangle ABO$ .



3. Find all positive integers  $a$  and  $b$  with  $a! + 4! = b^2$ .

4. For a fixed positive integer  $n$ , let  $x_1, x_2, \dots, x_n$  be  $n$  positive real numbers that sum to 1. Find the smallest possible value for the sum of the reciprocals of these  $n$  numbers.

5. Find all real numbers  $a$  such that the equation

$$|x - |x - |x - 4|| = a$$

has exactly three real solutions  $x$ . Here, of course,  $|x|$  is the absolute value of  $x$ .

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

<b>Return To</b>	<b>MATHEMATICS TALENT SEARCH</b> Dept. of Mathematics, Pomona College 610 N. College Ave., Claremont, CA 91711-6348	<b>Deadline</b> December 1, 2007	
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Team or Individual		5	