

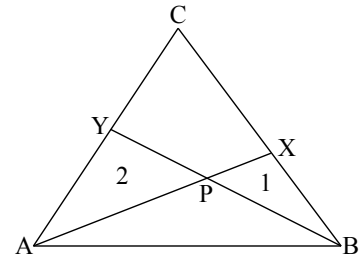
POMONA-WISCONSIN MATHEMATICS TALENT SEARCH

PROBLEM SET IV (2008-2009)

JANUARY 2009

1. (New Year's Problem.) Find all positive integers  $n$  such that  $n^2 + n + 2009$  is a square.

2. In the figure, the area of  $\triangle ABC$  is a whole number. Lines  $\overline{AX}$  and  $\overline{BY}$  are drawn, where  $X$  lies on side  $\overline{BC}$  and  $Y$  lies on side  $\overline{AC}$ , and these lines meet at point  $P$ , inside the triangle. The area of  $\triangle BPX$  is 1, the area of  $\triangle APY$  is 2, and the area of  $\triangle APB$  is a whole number. Find the area of  $\triangle ABC$ , and prove that your answer is correct.



3. Find a simple expression (in terms of  $n$ ) for the sum  $S_n$  of all of the numbers of the form  $k2^k$  where  $k$  is an integer and  $1 \leq k \leq n$ .

4. Decide (with proof) whether or not there exists a set  $\mathcal{E}$  of even positive integers such that every even positive integer can be written in a unique way as a sum of distinct members of  $\mathcal{E}$ . Similarly, decide if there exists a set  $\mathcal{O}$  of odd positive integers such that every odd positive integer can be written in a unique way as a sum of distinct members of  $\mathcal{O}$ .

5. Let  $n$  be a positive integer. A deck of  $2n$  numbered cards contains exactly two cards marked with each of the integers from 1 to  $n$ , and these are arranged in the order  $1, 1, 2, 2, 3, 3, \dots, n, n$  from top to bottom. Observe that if  $n = 3$ , the deck can be cut into two pieces, namely  $1, 1, 2, 2$  and  $3, 3$ , so that the sums of the numbers on the cards in the top and bottom parts are equal. Prove that there are infinitely many positive integers  $n$  for which the deck *cannot* be cut into two pieces so that the sums of the cards in the top and bottom parts are equal.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

<p><b>Return To</b></p> <p><b>Or Email To</b></p>	<p><b>MATHEMATICS TALENT SEARCH</b>                  Dept. of Mathematics, Pomona College                  610 N. College Ave., Claremont, CA 91711-6348                  mathsearch@pomona.edu                  emailed solutions require the information below too!</p>	<p><b>Deadline</b>                  February 2,                  2009</p>	
<p><b>Please Fill In</b></p>	<p><b>PROBLEM SET IV</b></p>	<p><b>Problem</b></p>	<p><b>Score</b></p>
<p>Name</p>		<p>1</p>	
<p>Home Address</p>		<p>2</p>	
<p>Town &amp; Zip</p>		<p>3</p>	
<p>School &amp; Grade</p>		<p>4</p>	
<p>Team or Individual</p>		<p>5</p>	