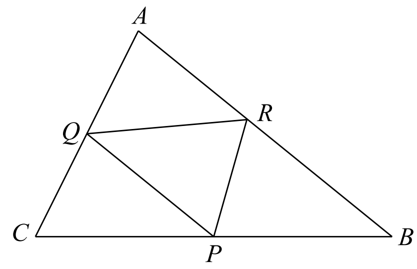
POMONA-WISCONSIN MATHEMATICS TALENT SEARCH

PROBLEM SET V (2007-2008) **FEBRUARY 2008**

1. Fix a real number $k > 0$. Let $x_1, x_2, \dots, x_n$ be n positive real numbers and suppose that $x_1(2k - x_2) = k^2$, $x_2(2k - x_3) = k^2$, $x_3(2k - x_4) = k^2, \dots$, and $x_n(2k - x_1) = k^2$. Show that all x_i are equal to k.

2. Given triangle $\triangle ABC$, let point P lie on side $\overline{BC}$, point Q on side $\overline{CA}$ and point R on side $\overline{AB}$. Suppose that the four triangles $\triangle ARQ$, $\triangle BPR$, $\triangle CQP$ and $\triangle PQR$ all have equal area. Show that P, Q and R are the midpoints of the respective sides.



3. Let d and n be positive integers, and let r be coprime to d. Show that for some integer k, the number $r + kd$ is coprime to n.

4. We call the squares of a 5 by 5 checkerboard adjacent if they share a side. What is the minimum number of squares that one can color yellow on such a 5 by 5 board so that every non-yellow square is adjacent to a yellow square?

5. Do there exist polynomials $P(x)$, $Q(x)$ and $R(x)$ having integer coefficients such that $P(x)$ is the product $P(x) = Q(x)R(x)$, all coefficients of $P(x)$ are 0, 1 or -1, and one of the coefficients of $Q(x)$ is 2008 ?

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

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Deadline March 7, 2008	
Problem	**Score**
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